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A singular one-parameter family of solutions in cubic superstring field theory

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Abstract: Performing a gauge transformation of a simple identity-like solution of superstring field theory, we construct a one-parameter family of solutions, and by evaluating the energy associated to this family, we show that for most of the values of the parameter the solution represents the tachyon vacuum, except for two isolated singular points where the solution becomes the perturbative vacuum and the half brane solution.

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1 Introduction

It is known that the analytic solutions for tachyon condensation [1–3] in open bosonic string field theory [4] as well as the ones [5–7] in cubic superstring field theory [8] are formally gauge equivalent to identity based solutions [9–13]. Identity based solutions are constructed as a product of certain linear combination of ghost number one operators with the identity string field [14–16].

Although identity based solutions are pathological solutions in the sense that they bring ambiguous analytic result for the value of the energy [17], by performing a gauge transformation over these solutions, it is possible to construct well behaved solutions. For instance, in reference [10], a one-parameter family of solutions has been found which interpolates between an identity based solution and the Erler-Schnabl's tachyon vacuum solution [2]. This result has been extended for the case of cubic superstring field theory [11], namely, a one-parameter family of solutions has been found which interpolates between an identity based solution and the Gorbachev's tachyon vacuum solution [7].

Motivated by the above results, and the recently discovered Erler's half brane solution [18] in cubic superstring field theory, in this paper, starting with the identity based solution [9, 12, 19, 20]

$$\widehat{\Phi}_I = \Big((c + B\gamma^2)(1 - K)\Big) \otimes \sigma_3, \tag{1.1}$$

by performing a gauge transformation of $\widehat{\Phi}_I$, we study the construction of the following one-parameter family of solutions

$$\widehat{\Phi}_\lambda = \Phi_{1,\lambda} \otimes \sigma_3 + \Phi_{2,\lambda} \otimes i\sigma_2, \tag{1.2}$$

where the string fields $\Phi_{1,\lambda}$ and $\Phi_{2,\lambda}$ are given by

$$\Phi_{1,\lambda} = Q(Bc)f(K,\lambda) + \lambda(2\lambda-1)cf(K,\lambda) + 4i\lambda(1-\lambda)cGBcG\widetilde{f}(K,\lambda), \tag{1.3}$$
$$\Phi_{2,\lambda} = Q(Bc)G\widetilde{f}(K,\lambda) + \lambda(2\lambda-1)cG\widetilde{f}(K,\lambda) + 4i\lambda(1-\lambda)cGBcf(K,\lambda), \tag{1.4}$$

with $f(K,\lambda)$ and $\widetilde{f}(K,\lambda)$ being functions of K,[1] and the parameter λ

$$f(K,\lambda) = \frac{\lambda^2(1-2\lambda)^2 + \left(16\lambda^3 - 32\lambda^2 + 18\lambda - 1\right)\lambda K}{\lambda^2(1-2\lambda)^2 + 2\lambda\left(8\lambda^3 - 16\lambda^2 + 10\lambda - 1\right)K + K^2}\,, \tag{1.5}$$
$$\widetilde{f}(K,\lambda) = \frac{4i(1-\lambda)\lambda K}{\lambda^2(1-2\lambda)^2 + 2\lambda\left(8\lambda^3 - 16\lambda^2 + 10\lambda - 1\right)K + K^2}\,. \tag{1.6}$$

Moreover, by explicit and detailed computation of the normalized value of the energy

$$E(\widehat{\Phi}_\lambda) = \frac{\pi^2}{3}\Big[\langle Y_{-2}\Phi_{1,\lambda}Q\Phi_{1,\lambda}\rangle + \langle Y_{-2}\Phi_{2,\lambda}Q\Phi_{2,\lambda}\rangle\Big] \tag{1.7}$$

associated to the solution $\widehat{\Phi}_\lambda$, we obtain

$$E(\widehat{\Phi}_\lambda) = \begin{cases} 0, & \lambda = 0\,,\ \text{ Perturbative Vacuum Solution,} \\ -1/2, & \lambda = 1/2\,,\ \text{ Half Brane Solution,} \\ -1, & \big(\lambda<0\big) \vee \big(\kappa \le \lambda < \tfrac{1}{2}\big) \vee \big(\lambda > \tfrac{1}{2}\big)\,,\ \text{ Tachyon Vacuum Solution,} \end{cases} \tag{1.8}$$

where κ is a numerical constant defined as

$$\kappa = \frac{2}{3} - \frac{1}{6}\left(\frac{25}{2} + \frac{3}{2}\sqrt{69}\right)^{1/3} - \frac{1}{6}\left(\frac{25}{2} - \frac{3}{2}\sqrt{69}\right)^{1/3} \approx 0.122561. \tag{1.9}$$

Note that for most of the values of the parameter λ, the solution represents the tachyon vacuum, while the two isolated points $\lambda = 0$ and $\lambda = 1/2$ correspond to the perturbative vacuum and the half brane solution respectively.

We expect that the construction of a one-parameter family of solutions using identity based solutions, in cubic superstring field theory, will provide us with relevant tools to analyze other important solutions, such as the multibrane solutions [19, 20], and the recently proposed Erler's analytic solution for tachyon condensation in Berkovits non-polynomial open superstring field theory [24]. Since the algebraic structure of Berkovits theory [25] is similar to the cubic superstring field theory, the results of our work can be naturally extended, however, the presence of a non-polynomial action in Berkovits theory will bring us challenges in the search of new solutions.

This paper is organized as follows. In section 2, we review the modified cubic superstring field theory and introduce some notations and conventions. Since the explicit

[1] The K field is an element of the so-called KBc subalgebra introduced in the references [3, 21–23].

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form of our one-parameter family of solutions is expressed in terms of elements of the $GKBc\gamma$ algebra, in section 3, we study in detail this algebra. In section 4, by performing a gauge transformation of an identity based solution, we show the construction of the one-parameter family of solutions. In section 5, we analyze correlation functions involving the G field and as a pedagogical application of these correlators, we show the computation of the energy for the half brane solution. In section 6, we evaluate the energy associated to the one-parameter family of solutions. In section 7, a summary and further directions of exploration are given.

2 Modified cubic superstring field theory, notations and conventions

The action of the modified cubic superstring field theory which takes into account the $GSO(+)$ and $GSO(-)$ sectors is given by [8]

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle Y_{-2}\Phi_1 Q\Phi_1\rangle + \frac{1}{3}\langle Y_{-2}\Phi_1\Phi_1\Phi_1\rangle + \frac{1}{2}\langle Y_{-2}\Phi_2 Q\Phi_2\rangle - \langle Y_{-2}\Phi_1\Phi_2\Phi_2\rangle\right], \quad (2.1)$$

where Q is the BRST operator of the open Neveu-Schwarz superstring theory. The operator Y_{-2} is inserted at the open string midpoint and is written as the product of two inverse picture changing operators $Y_{-2} = Y(i)Y(-i)$, where $Y(z) = -\partial\xi e^{-2\phi}c(z)$. The ghost number one string fields Φ_1 and Φ_2 belong to the $GSO(+)$ and $GSO(-)$ sectors, and are Grassman odd and Grassman even respectively.

Varying the action (2.1) with respect to the string fields Φ_1 and Φ_2 yields the following equations of motion [26]

$$Q\Phi_1 + \Phi_1 * \Phi_1 - \Phi_2 * \Phi_2 = 0, \quad (2.2)$$

$$Q\Phi_2 + \Phi_1 * \Phi_2 - \Phi_2 * \Phi_1 = 0. \quad (2.3)$$

Regarding to the star product, we are going to use the left handed convention of [1, 2]. There are other sources which use the right handed convention [3, 27], for details related to the connection between these two conventions see reference [18].

Using the equations of motion (2.2), (2.3) and the cyclicity relation

$$\langle Y_{-2}\Phi_1\Phi_2\Phi_2\rangle = -\langle Y_{-2}\Phi_2\Phi_1\Phi_2\rangle = \langle Y_{-2}\Phi_2\Phi_2\Phi_1\rangle, \quad (2.4)$$

where an additional minus sign arises due to the fact that Φ_2 belongs to the $GSO(-)$ sector,[2] we can write the action (2.1) as

$$S = -\frac{1}{6g^2}\Big[\langle Y_{-2}\Phi_1 Q\Phi_1\rangle + \langle Y_{-2}\Phi_2 Q\Phi_2\rangle\Big]. \quad (2.5)$$

Since Φ_2 has opposite Grassmannality as compared to the $GSO(+)$ string field Φ_1, it seems that they fail to obey common algebraic relations. This problem can be resolved

[2] Since a string field belonging to the $GSO(-)$ sector has half-integer conformal weight, Φ_2 changes its sign under the conformal transformation $\mathcal{R}_{2\pi}$ representing the 2π rotation of the unit disk [28].

by attaching the 2×2 internal Chan-Paton matrices to the string fields and the operator insertions as [27, 29]

$$\widehat{Q} = Q \otimes \sigma_3, \quad \widehat{Y}_{-2} = Y_{-2} \otimes \sigma_3, \tag{2.6}$$

$$\widehat{\Phi} = \Phi_1 \otimes \sigma_3 + \Phi_2 \otimes i\sigma_2. \tag{2.7}$$

Using these definitions, the action (2.1) can be written in a compact way

$$S = -\frac{1}{2g^2}\mathrm{Tr}\left[\frac{1}{2}\langle \widehat{Y}_{-2}\widehat{\Phi}\widehat{Q}\widehat{\Phi}\rangle + \frac{1}{3}\langle \widehat{Y}_{-2}\widehat{\Phi}\widehat{\Phi}\widehat{\Phi}\rangle\right], \tag{2.8}$$

and the equations of motion (2.2) and (2.3) are reduced to a single equation

$$\widehat{Q}\widehat{\Phi} + \widehat{\Phi}\widehat{\Phi} = 0. \tag{2.9}$$

For a given ghost number zero string field $\widehat{U} = U_1 \otimes \mathbb{I} + U_2 \otimes \sigma_1$, we can construct a gauge transformation of the string field $\widehat{\Phi}$ as follows

$$\widehat{\Psi} = \widehat{U}(\widehat{Q} + \widehat{\Phi})\widehat{U}^{-1}. \tag{2.10}$$

It turns out that the action (2.8) is invariant under this gauge transformation (2.10). If $\widehat{\Phi}$ is a solution of the equation of motion (2.9) then a string field $\widehat{\Psi}$, related to $\widehat{\Phi}$ by means of the equation (2.10), is also a solution.

In order to find analytic solutions of the equation of motion (2.9), we can employ the prescription studied in reference [9], namely, (i) find a simplest identity based solution of the equation of motion,[3] (ii) perform a gauge transformation over this identity based solution such that the resulting string field, consistently, represents a well behaved solution [10, 11].

In this paper, following the above procedures, we are going to construct a one-parameter family of solutions $\widehat{\Phi}_\lambda$ and evaluate the energy associated to these solutions. It turns out that, depending on the value of the parameter λ, the solutions $\widehat{\Phi}_\lambda$ describe three distinct gauge orbits corresponding to the perturbative vacuum, the half brane and the tachyon vacuum solution. Before deriving the explicit form of the solution $\widehat{\Phi}_\lambda$, in the next section we will introduce the so-called $GKBc\gamma$ algebra.

3 The $GKBc\gamma$ algebra, definitions and star products

The $GKBc\gamma$ algebra is an extension of the well known $KBc\gamma$ algebra [5, 21, 22]. Essentially, we add the new element G to the $KBc\gamma$ algebra. This string field G lives in the $GSO(-)$ sector, and is related to the worldsheet supercurrent $G(z)$ [18].

To derive some identities involving the star product of the basic string fields G, K, B, c and γ together with the action of the BRST operator Q over elements of the $GKBc\gamma$ algebra, it will be useful to write the following representation of these fields in terms of

[3] Although the identity based solution formally satisfies the equation of motion (2.9), it is a pathological solution in the sense that it brings ambiguous analytic result for the value of the energy [9, 14–16].

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operators acting on the identity string field $|I\rangle = U_1^\dagger U_1|0\rangle$

$$K \equiv \frac{1}{2}\hat{\mathcal{L}}U_1^\dagger U_1|0\rangle, \tag{3.1}$$

$$B \equiv \frac{1}{2}\hat{\mathcal{B}}U_1^\dagger U_1|0\rangle, \tag{3.2}$$

$$G \equiv \frac{1}{2}\hat{\mathcal{G}}U_1^\dagger U_1|0\rangle, \tag{3.3}$$

$$c \equiv U_1^\dagger U_1\tilde{c}(0)|0\rangle, \tag{3.4}$$

$$\gamma \equiv U_1^\dagger U_1\tilde{\gamma}(0)|0\rangle. \tag{3.5}$$

The operators $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$, $\hat{\mathcal{G}}$, $\tilde{c}(0)$ and $\tilde{\gamma}(0)$ are defined in the sliver frame ($\tilde{z}$ coordinate),[4] and they are related to the worldsheet energy-momentum tensor, the b field, the worldsheet supercurrent, the c and γ ghosts fields respectively, for instance

$$\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger = \oint \frac{dz}{2\pi i}(1+z^2)(\arctan z + \mathrm{arccot} z)\, T(z)\,, \tag{3.6}$$

$$\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger = \oint \frac{dz}{2\pi i}(1+z^2)(\arctan z + \mathrm{arccot} z)\, b(z)\,, \tag{3.7}$$

$$\hat{\mathcal{G}} \equiv \mathcal{G}_{1/2} + \mathcal{G}_{1/2}^\dagger = \sqrt{\frac{2}{\pi}}\oint \frac{dz}{2\pi i}(1+z^2)^{1/2}(\arctan z + \mathrm{arccot} z)\, G(z)\,, \tag{3.8}$$

while the operator $U_1^\dagger U_1$ in general is given by $U_r^\dagger U_r = e^{\frac{2-r}{2}\hat{\mathcal{L}}}$, so we have chosen $r=1$, note that the string field $U_1^\dagger U_1|0\rangle$ represents to the identity string field. To compute star products of string fields involving the operators $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$ and $\hat{\mathcal{G}}$, it should be useful to define the operators

$$\mathcal{L}_{-1} \equiv \frac{\pi}{2}\oint \frac{dz}{2\pi i}(1+z^2)T(z) = \frac{\pi}{2}(L_{-1}+L_1)\,, \tag{3.9}$$

$$\mathcal{B}_{-1} \equiv \frac{\pi}{2}\oint \frac{dz}{2\pi i}(1+z^2)b(z) = \frac{\pi}{2}(b_{-1}+b_1)\,, \tag{3.10}$$

$$\mathcal{G}_{-1/2} \equiv \sqrt{\frac{\pi}{2}}\oint \frac{dz}{2\pi i}(1+z^2)^{1/2}G(z)\,. \tag{3.11}$$

Given two string fields ϕ and φ belonging to the $GSO(+)$ or the $GSO(-)$ sector, we can show that

$$(\hat{\mathcal{B}}\phi) * \varphi = \hat{\mathcal{B}}(\phi * \varphi) + (-1)^{\mathrm{gn}(\phi)}\phi * \mathcal{B}_{-1}\varphi\,, \tag{3.12}$$

$$\phi * (\hat{\mathcal{B}}\varphi) = (-1)^{\mathrm{gn}(\phi)}\hat{\mathcal{B}}(\phi * \varphi) - (-1)^{\mathrm{gn}(\phi)}(\mathcal{B}_{-1}\phi) * \varphi\,, \tag{3.13}$$

$$(\hat{\mathcal{B}}\phi) * (\hat{\mathcal{B}}\varphi) = -(-1)^{\mathrm{gn}(\phi)}\hat{\mathcal{B}}\mathcal{B}_{-1}(\phi * \varphi) + (\mathcal{B}_{-1}\phi) * (\mathcal{B}_{-1}\varphi)\,, \tag{3.14}$$

$$(\hat{\mathcal{G}}\phi) * \varphi = \hat{\mathcal{G}}(\phi * \varphi) + (-1)^{\mathrm{gn}(\phi)}\phi * \mathcal{G}_{-1/2}\varphi\,, \tag{3.15}$$

$$\phi * (\hat{\mathcal{G}}\varphi) = (-1)^{\mathrm{gn}(\phi)}\hat{\mathcal{G}}(\phi * \varphi) - (-1)^{\mathrm{gn}(\phi)}(\mathcal{G}_{-1/2}\phi) * \varphi\,, \tag{3.16}$$

[4] To map a point z in the upper half plane to a point $\tilde{z}$ in the sliver frame, we are using the conformal transformation $\tilde{z} = \frac{2}{\pi}\arctan z$ [2]. There is another convention for the conformal transformation which is given by $\tilde{z} = \arctan z$ [1]. In this convention, instead of the factor 1/2 in front of the R.H.S. of equations (3.1)–(3.3), we should have the factor $1/\pi$.

$$
\begin{aligned}
(\hat{\mathcal{G}}\phi) * (\hat{\mathcal{G}}\varphi) &= -(-1)^{\mathrm{gn}(\phi)}\hat{\mathcal{G}}\mathcal{G}_{-1/2}(\phi * \varphi) + (\mathcal{G}_{-1/2}\phi) * (\mathcal{G}_{-1/2}\varphi) \\
&\quad +(-1)^{\mathrm{gn}(\phi)}2\hat{\mathcal{L}}(\phi * \varphi) + (-1)^{\mathrm{gn}(\phi)}\phi * \mathcal{L}_{-1}\varphi \\
&\quad -(-1)^{\mathrm{gn}(\phi)}(\mathcal{L}_{-1}\phi) * \varphi \,, \qquad (3.17)\\
(\hat{\mathcal{L}}^n\phi) * \varphi &= \sum_{n'=0}^{n} \binom{n}{n'} \hat{\mathcal{L}}^{n-n'}(\phi * \mathcal{L}_{-1}^{n'}\varphi) \,, \qquad (3.18)\\
\phi * (\hat{\mathcal{L}}^n\varphi) &= \sum_{n'=0}^{n} \binom{n}{n'} (-1)^{n'} \hat{\mathcal{L}}^{n-n'}((\mathcal{L}_{-1}^{n'}\phi) * \varphi) \,, \qquad (3.19)\\
(\hat{\mathcal{L}}^m\phi) * (\hat{\mathcal{L}}^n\varphi) &= \sum_{m'=0}^{m}\sum_{n'=0}^{n} \binom{m}{m'}\binom{n}{n'} (-1)^{n'} \hat{\mathcal{L}}^{m+n-m'-n'}((\mathcal{L}_{-1}^{n'}\phi) * (\mathcal{L}_{-1}^{m'}\varphi)) \,, \qquad (3.20)
\end{aligned}
$$

where $\mathrm{gn}(\phi)$ takes into account the Grassmannality of the string field ϕ. The above results, containing the operator $\hat{\mathcal{G}}$, are new and they are an extension of the result derived in [1].

Regarding the wedge states with insertions, the star product of two of them is written in the form

$$
U_r^\dagger U_r \tilde{\phi}(\tilde{x})|0\rangle * U_s^\dagger U_s \tilde{\psi}(\tilde{y})|0\rangle = U_t^\dagger U_t \tilde{\phi}\Big(\tilde{x} + \frac{1}{2}(s-1)\Big)\tilde{\psi}\Big(\tilde{y} - \frac{1}{2}(r-1)\Big)|0\rangle, \qquad (3.21)
$$

where $t = r + s - 1$, and by $\tilde{\phi}(\tilde{x})$ we denote a local operator $\phi(z)$ expressed in the sliver frame, which in the special case of primary field with conformal weight h is given by

$$
\tilde{\phi}(\tilde{z}) = \left(\frac{dz}{d\tilde{z}}\right)^h \phi(z) = \left(\frac{\pi}{2}\right)^h \cos^{-2h}\left(\frac{\pi\tilde{z}}{2}\right)\phi\bigg(\tan\left(\frac{\pi\tilde{z}}{2}\right)\bigg). \qquad (3.22)
$$

Since we are using the conformal transformation $\tilde{z} = \frac{2}{\pi}\arctan z$ which is a bit different from the one used in Schnabl's original paper $\tilde{z} = \arctan z$ [1], we have a factor $1/2$ in the R.H.S. of equation (3.21) instead of the factor $\pi/4$ which is present in the reference [1].

It will be useful to know the action of the BRST, $\mathcal{L}_{-1}$, $\mathcal{B}_{-1}$ and $\mathcal{G}_{-1/2}$ operators on the star product of two string fields

$$
\begin{aligned}
Q(\phi * \varphi) &= (Q\phi) * \varphi + (-1)^{\mathrm{gn}(\phi)}\phi * (Q\varphi), \qquad (3.23)\\
\mathcal{L}_{-1}(\phi * \varphi) &= (\mathcal{L}_{-1}\phi) * \varphi + \phi * (\mathcal{L}_{-1}\varphi), \qquad (3.24)\\
\mathcal{B}_{-1}(\phi * \varphi) &= (\mathcal{B}_{-1}\phi) * \varphi + (-1)^{\mathrm{gn}(\phi)}\phi * (\mathcal{B}_{-1}\varphi), \qquad (3.25)\\
\mathcal{G}_{-1/2}(\phi * \varphi) &= (\mathcal{G}_{-1/2}\phi) * \varphi + (-1)^{\mathrm{gn}(\phi)}\phi * (\mathcal{G}_{-1/2}\varphi)\,. \qquad (3.26)
\end{aligned}
$$

Let us derive the algebra associated to the set of operators defined by equations (3.1)–(3.5). As a pedagogical illustration, we explicitly compute the product G^2

$$
G^2 \equiv G * G = \frac{1}{2}\hat{\mathcal{G}}U_1^\dagger U_1|0\rangle * \frac{1}{2}\hat{\mathcal{G}}U_1^\dagger U_1|0\rangle = \frac{1}{4}\hat{\mathcal{G}}U_1^\dagger U_1|0\rangle * \hat{\mathcal{G}}U_1^\dagger U_1|0\rangle, \qquad (3.27)
$$

using equation (3.17) and the commutators $[\mathcal{G}_{-1/2}, \hat{\mathcal{L}}] = 0$, $[\mathcal{L}_{-1}, \hat{\mathcal{L}}] = 0$, we obtain

$$
G^2 = \frac{2}{4}\hat{\mathcal{L}}\Big(U_1^\dagger U_1|0\rangle * U_1^\dagger U_1|0\rangle\Big) = \frac{1}{2}\hat{\mathcal{L}}U_1^\dagger U_1|0\rangle, \qquad (3.28)
$$

therefore we have that $G^2 = K$.

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Following the same steps, using equations (3.12)–(3.20), the commutator relation $[\mathcal{G}_{-1/2}, \tilde{\gamma}(0)] = -\frac{1}{2}\partial\tilde{c}(0)$ and the anti-commutator $\{\mathcal{G}_{-1/2}, \tilde{c}(0)\} = -2\tilde{\gamma}(0)$, we can show that

$$\{G, G\} = 2K, \qquad [K, B] = 0, \qquad [K, G] = 0, \qquad \{B, G\} = 0, \tag{3.29}$$

$$\partial c = [K, c], \qquad \partial\gamma = [K, \gamma], \qquad B^2 = 0, \qquad c^2 = 0, \tag{3.30}$$

$$\{G, c\} = -2\gamma, \qquad [G, \gamma] = -\frac{1}{2}\partial c\,, \tag{3.31}$$

where the expressions ∂c and $\partial\gamma$ have been defined as $\partial\phi \equiv U_1^\dagger U_1 \partial\tilde{\phi}(0)|0\rangle$.

The action of the BRST operator Q on the basic string fields K, G, B, c and γ is given by

$$QK = 0, \quad QG = 0, \quad QB = K, \tag{3.32}$$

$$Qc = cKc - \gamma^2, \tag{3.33}$$

$$Q\gamma = c\partial\gamma - \frac{1}{2}\gamma\partial c\,. \tag{3.34}$$

Now we are in position to study and present the construction of a one-parameter family of solutions.

4 One-parameter family of solutions from an identity based solution

It is known that a solution to the equation of motion (2.9) is given by the following simplest identity based solution [9, 12, 19, 20]

$$\widehat{\Phi}_I = \Big((c + B\gamma^2)(1 - K)\Big) \otimes \sigma_3. \tag{4.1}$$

Using this identity based solution (4.1), we will show that it is possible to construct a one-parameter family of solutions $\widehat{\Phi}_\lambda$ which depending on the value of the parameter λ will describe three distinct gauge orbits corresponding to the perturbative vacuum, the half brane and the tachyon vacuum solution.

Let us write the explicit form of the aforementioned gauge transformation

$$\widehat{\Phi}_\lambda = \widehat{U}_\lambda(\widehat{Q} + \widehat{\Psi}_I)\widehat{U}_\lambda^{-1}, \tag{4.2}$$

$\widehat{U}_\lambda$ is a ghost number zero string field given by[5]

$$\widehat{U}_\lambda = \Big(1 + cB[K + (\lambda - 1)(2\lambda + 1)]\Big) \otimes \mathbb{I} + 4i\lambda(1 - \lambda)cBG \otimes \sigma_1, \tag{4.4}$$

$$\widehat{U}_\lambda^{-1} = \left(1 - cB\frac{K - 1 + f(K, \lambda)}{K}\right) \otimes \mathbb{I} - c\frac{\widetilde{f}(K, \lambda)}{K}BG \otimes \sigma_1, \tag{4.5}$$

[5]We would like to bring few motivational words explaining the choice (4.4). As in the bosonic case [9], for superstring field theory we can also construct a gauge transformation which relates the identity based solution (4.1) with the half brane solution [18]. The gauge transformation which does this job precisely corresponds to a $\widehat{U}$ given by

$$\widehat{U} = \Big(1 + cB[K - 1]\Big) \otimes \mathbb{I} + icBG \otimes \sigma_1. \tag{4.3}$$

Applying a supersymmetric analog of the Zeze map [10], we consider a slight modification of (4.3) in which a real parameter λ is inserted in the cBK and cBG pieces in the gauge transformation such that for $\lambda = 0$ and $\lambda = 1$, we recover the perturbative and tachyon vacua respectively.

where $f(K,\lambda)$ and $\widetilde{f}(K,\lambda)$ are the following functions

$$f(K,\lambda) = \frac{\lambda^2(1-2\lambda)^2 + \left(16\lambda^3 - 32\lambda^2 + 18\lambda - 1\right)\lambda K}{\lambda^2(1-2\lambda)^2 + 2\lambda\left(8\lambda^3 - 16\lambda^2 + 10\lambda - 1\right)K + K^2}\,, \tag{4.6}$$

$$\widetilde{f}(K,\lambda) = \frac{4i(1-\lambda)\lambda K}{\lambda^2(1-2\lambda)^2 + 2\lambda\left(8\lambda^3 - 16\lambda^2 + 10\lambda - 1\right)K + K^2}\,. \tag{4.7}$$

Then, the one-parameter family of solutions is obtained by performing the above gauge transformation over the identity based solution (4.1)

$$\begin{aligned}\widehat{\Phi}_\lambda &= \widehat{U}_\lambda \widehat{Q} \widehat{U}_\lambda^{-1} + \widehat{U}_\lambda\Big((c+B\gamma^2)(1-K)\otimes\sigma_3\Big)\widehat{U}_\lambda^{-1} \\ &= \Phi_{1,\lambda}\otimes\sigma_3 + \Phi_{2,\lambda}\otimes i\sigma_2,\end{aligned} \tag{4.8}$$

where the string fields $\Phi_{1,\lambda}$ and $\Phi_{2,\lambda}$ are given by

$$\Phi_{1,\lambda} = Q(Bc)f(K,\lambda) + \lambda(2\lambda-1)cf(K,\lambda) + 4i\lambda(1-\lambda)cGBcG\widetilde{f}(K,\lambda), \tag{4.9}$$

$$\Phi_{2,\lambda} = Q(Bc)G\widetilde{f}(K,\lambda) + \lambda(2\lambda-1)cG\widetilde{f}(K,\lambda) + 4i\lambda(1-\lambda)cGBcf(K,\lambda). \tag{4.10}$$

A check of the equation of motion for the above solution is straightforward.

At this point we can ask about the interval where the parameter λ should belong, the answer to this question will be studied later, for the time being, let us analyze the solution for particular values of this parameter.

For the value of the parameter $\lambda = 0$, we identically obtain $\widehat{\Phi}_{\lambda=0} = 0$ and thus this case corresponds to the perturbative vacuum.

For the value $\lambda = 1$, we see that $\widetilde{f}(K,\lambda=1) = 0$ and $f(K,\lambda=1) = 1/(1+K)$, therefore we obtain

$$\widehat{\Phi}_{\lambda=1} = \big[Q(Bc) + c\big]\frac{1}{1+K}\otimes\sigma_3. \tag{4.11}$$

This solution precisely represents the tachyon vacuum solution. The energy of this solution (4.11) has been evaluated in references [7, 9] given a result in agreement with Sen's first conjecture.

For the value $\lambda = 1/2$, we get $\widetilde{f}(K,\lambda=1/2) = i/(1+K)$ and $f(K,\lambda=1/2) = 1/(1+K)$, so in this case the solution can be written as

$$\widehat{\Phi}_{\lambda=1/2} = \Big[Q(Bc) - cGBcG\Big]\frac{1}{1+K}\otimes\sigma_3 + \Big[iQ(Bc)G + icGBc\Big]\frac{1}{1+K}\otimes i\sigma_2. \tag{4.12}$$

This solution has been studied in reference [18] and since the evaluation of its energy brings a result which is half of the value of the tachyon vacuum energy, the solution (4.12) has been called as the half brane solution.

Note that to recognize the kind of solution we have, we must calculate the energy associated to the solution. For any solution of the form $\widehat{\Phi} = \Phi_1\otimes\sigma_3 + \Phi_2\otimes i\sigma_2$, employing equation (2.5), we can write the normalized value of the energy E as follows

$$E(\widehat{\Phi}) \equiv -2\pi^2 g^2 S = \frac{\pi^2}{3}\Big[\langle Y_{-2}\Phi_1 Q\Phi_1\rangle + \langle Y_{-2}\Phi_2 Q\Phi_2\rangle\Big]. \tag{4.13}$$

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To evaluate the energy (4.13) for the solution (4.8) with a generic value of the parameter λ, we will require to define and study correlation functions involving elements of the $GKBc\gamma$ algebra. In the next section, we are going to consider correlation functions including the G field and as a pedagogical application of these correlators, we will show the computation of the energy for the half brane solution.

5 Correlation functions and the half brane energy

To compute the energy for solutions constructed out of elements of the $GKBc\gamma$ algebra, it will be useful to know correlation functions defined on a semi-infinite cylinder of circumference l denoted by C_l.

A point z on the upper half-plane can be mapped to a point $\tilde{z} \in C_l$, which has the property that $\tilde{z} \simeq \tilde{z} + l$, through the conformal transformation

$$\tilde{z} = \frac{l}{\pi} \arctan z, \tag{5.1}$$

The expression for the conformal transformation of primary fields with conformal weight h is given by

$$\tilde{\phi}(\tilde{z}) = \left(\frac{dz}{d\tilde{z}}\right)^h \phi(z) = \left(\frac{\pi}{l}\right)^h \cos^{-2h}\left(\frac{\pi\tilde{z}}{l}\right)\phi\left(\tan\left(\frac{\pi\tilde{z}}{l}\right)\right). \tag{5.2}$$

Using (5.1) and (5.2), we can derive the following correlation function involving the $b(z)$, $c(z)$ and $\gamma(z)$ ghost fields

$$\langle Y_{-2} c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \frac{l^2}{2\pi^2}\cos\left(\frac{\pi(\tilde{y}-\tilde{z})}{l}\right), \tag{5.3}$$

$$\langle Y_{-2} b(\tilde{v})c(\tilde{w})c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \frac{l\csc\left(\frac{\pi(\tilde{v}-\tilde{w})}{l}\right)\csc\left(\frac{\pi(\tilde{v}-\tilde{x})}{l}\right)\sin\left(\frac{\pi(\tilde{w}-\tilde{x})}{l}\right)\cos\left(\frac{\pi(\tilde{y}-\tilde{z})}{l}\right)}{2\pi}. \tag{5.4}$$

Using (5.4), let us compute the correlator $\langle Y_{-2} Bc(\tilde{w})c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l}$. Since the B field can be defined as a line integral insertion of the $b(z)$ ghost field inside correlation functions on the cylinder [3], we can write

$$\langle Y_{-2} Bc(\tilde{w})c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \langle Y_{-2}\int_{-i\infty}^{i\infty}\frac{d\tilde{v}}{2\pi i} b(\tilde{v})c(\tilde{w})c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l}. \tag{5.5}$$

Plugging (5.4) into the R.H.S. of equation (5.5) and employing the integral

$$\int_{-i\infty}^{i\infty} d\tilde{v}\,\csc\left(\frac{\pi(\tilde{v}-\tilde{w})}{l}\right)\csc\left(\frac{\pi(\tilde{v}-\tilde{x})}{l}\right) = 2i(\tilde{w}-\tilde{x})\csc\left(\frac{\pi(\tilde{w}-\tilde{x})}{l}\right), \tag{5.6}$$

we obtain

$$\langle Y_{-2} Bc(\tilde{w})c(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \frac{l}{2\pi^2}(\tilde{w}-\tilde{x})\cos\left(\frac{\pi(\tilde{y}-\tilde{z})}{l}\right). \tag{5.7}$$

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In the same way, by writing the G field as a line integral insertion of the worldsheet supercurrent $G(z)$ inside correlation functions on the cylinder, we can derive the following correlators

$$\langle Y_{-2}Gc(\tilde{x})c(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \frac{l^2}{2\pi^2}\left[\cos\left(\frac{\pi\,(\tilde{y}-\tilde{z})}{l}\right) - \cos\left(\frac{\pi\,(\tilde{x}-\tilde{z})}{l}\right)\right], \tag{5.8}$$

$$\langle Y_{-2}GBc(\tilde{w})c(\tilde{x})c(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \\ = \frac{l\left((\tilde{x}-\tilde{y})\cos\left(\frac{\pi(\tilde{w}-\tilde{z})}{l}\right) + (\tilde{y}-\tilde{w})\cos\left(\frac{\pi(\tilde{x}-\tilde{z})}{l}\right) + (\tilde{w}-\tilde{x})\cos\left(\frac{\pi(\tilde{y}-\tilde{z})}{l}\right)\right)}{2\pi^2}, \tag{5.9}$$

$$\langle Y_{-2}GBc(\tilde{w})\gamma(\tilde{x})\gamma(\tilde{y})\gamma(\tilde{z})\rangle_{C_l} = \frac{l\left(\cos\left(\frac{\pi(\tilde{x}-\tilde{y})}{l}\right) + \cos\left(\frac{\pi(\tilde{x}-\tilde{z})}{l}\right) + \cos\left(\frac{\pi(\tilde{y}-\tilde{z})}{l}\right)\right)}{8\pi^2}. \tag{5.10}$$

With the aid of these correlation functions, we are ready to evaluate the energy associated to the half brane solution. Using equation (4.13) for the particular case of the solution (4.12), and noting that the BRST exact terms do not contribute to the evaluation of the energy, we obtain

$$E(\widehat{\Phi}_{\lambda=1/2}) = \frac{\pi^2}{3}\left[\left\langle\!\!\left\langle cGBcG\frac{1}{1+K}Q(cGBc)G\frac{1}{1+K}\right\rangle\!\!\right\rangle - \left\langle\!\!\left\langle cGBc\frac{1}{1+K}Q(cGBc)\frac{1}{1+K}\right\rangle\!\!\right\rangle\right], \tag{5.11}$$

where the notation $\langle\langle\cdots\rangle\rangle$ means that $\langle\langle\cdots\rangle\rangle \equiv \langle Y_{-2}\cdots\rangle$. Employing equations (3.29)–(3.34), after a lengthy algebraic manipulations, from equation (5.11) we arrive to

$$\begin{aligned}
E(\widehat{\Phi}_{\lambda=1/2}) = \frac{\pi^2}{3}\Bigg[&\left\langle\!\!\left\langle Kc\frac{1}{1+K}\gamma^2\frac{1}{1+K}\right\rangle\!\!\right\rangle + 3\left\langle\!\!\left\langle KcK\frac{1}{1+K}\gamma^2\frac{1}{1+K}\right\rangle\!\!\right\rangle \\
&-\frac{2}{3}\left\langle\!\!\left\langle GcK^2\frac{1}{1+K}c\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle + \left\langle\!\!\left\langle G\gamma\frac{1}{1+K}cKc\frac{1}{1+K}\right\rangle\!\!\right\rangle - 5\left\langle\!\!\left\langle BcKc\frac{1}{1+K}\gamma^2\frac{1}{1+K}\right\rangle\!\!\right\rangle \\
&-4\left\langle\!\!\left\langle Bc\gamma K\frac{1}{1+K}c\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle + 2\left\langle\!\!\left\langle Bc\gamma K^2\frac{1}{1+K}c\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle + 4\left\langle\!\!\left\langle GBc\gamma\frac{1}{1+K}\gamma^2\frac{1}{1+K}\right\rangle\!\!\right\rangle \\
&-6\left\langle\!\!\left\langle Bc\gamma K\frac{1}{1+K}c\gamma K\frac{1}{1+K}\right\rangle\!\!\right\rangle + 4\left\langle\!\!\left\langle GBcK\frac{1}{1+K}\gamma^3\frac{1}{1+K}\right\rangle\!\!\right\rangle \\
&-3\left\langle\!\!\left\langle GBc\frac{1}{1+K}cKc\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle - 3\left\langle\!\!\left\langle GBcK\frac{1}{1+K}cKc\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle\Bigg]
\end{aligned} \tag{5.12}$$

All the above correlators can be computed using equations (5.3) and (5.7)–(5.10), for instance, let us explicitly compute the correlator $\left\langle\!\left\langle GBcK\frac{1}{1+K}cKc\gamma\frac{1}{1+K}\right\rangle\!\right\rangle$

$$\left\langle\!\!\left\langle GBcK\frac{1}{1+K}cKc\gamma\frac{1}{1+K}\right\rangle\!\!\right\rangle = \int_0^\infty dt_1 dt_2\, e^{-t_1-t_2}\partial_{s_1}\partial_{s_2}\left[\langle\langle GBc\Omega^{s_1+t_1}c\Omega^{s_2}c\gamma\Omega^{t_2}\rangle\rangle\right]\Big|_{s_1=s_2=0}, \tag{5.13}$$

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where we have used the fact that $\Omega^t = e^{-tK}$. The correlator $\langle\langle GBc\Omega^{s_1+t_1}c\Omega^{s_2}c\gamma\Omega^{t_2}\rangle\rangle$ is given by

$$\langle\langle GBc\Omega^{s_1+t_1}c\Omega^{s_2}c\gamma\Omega^{t_2}\rangle\rangle = \langle Y_{-2}GBc(s_1+s_2+t_1+t_2)c(s_2+t_2)c(t_2)\gamma(t_2)\rangle_{C_{s_1+s_2+t_1+t_2}}. \tag{5.14}$$

The R.H.S. of equation (5.14) can be evaluated using equation (5.9), so that we obtain the result

$$\partial_{s_1}\partial_{s_2}\Big[\langle\langle GBc\Omega^{s_1+t_1}c\Omega^{s_2}c\gamma\Omega^{t_2}\rangle\rangle\Big]\Big|_{s_1=s_2=0} = \\ = \frac{t_1\left(\cos\left(\frac{\pi t_1}{t_1+t_2}\right)-1\right)+t_2\left(-\pi\sin\left(\frac{\pi t_1}{t_1+t_2}\right)+\cos\left(\frac{\pi t_1}{t_1+t_2}\right)-1\right)}{2\pi^2\,(t_1+t_2)}. \tag{5.15}$$

Performing the change of variables $t_1 \to uv$, $t_2 \to u-uv$, $\int_0^\infty dt_1 dt_2 \to \int_0^\infty du \int_0^1 dv\, u$, and using the result (5.15), from equation (5.13), we get

$$\langle\langle GBcK\frac{1}{1+K}cKc\gamma\frac{1}{1+K}\rangle\rangle = \int_0^\infty du\int_0^1 dv\,\frac{e^{-u}u(\pi(v-1)\sin(\pi v)+\cos(\pi v)-1)}{2\pi^2} \\ = -\frac{1}{\pi^2}\,. \tag{5.16}$$

Performing similar computations for the rest of terms appearing on the R.H.S. of equation (5.12) and adding the results up, the energy turns out to be

$$E(\widehat{\Phi}_{\lambda=1/2}) = \frac{\pi^2}{3}\left[-\frac{3}{2\pi^2}\right] = -\frac{1}{2}, \tag{5.17}$$

this is precisely $1/2$ times the normalized value of the tachyon vacuum energy which has the value $E(\widehat{\Phi}_{\lambda=1}) = -1$.

Let us summarize the results for the normalized value of the energy (4.13) which has been obtained for the particular values of the parameter $\lambda = \{0, 1/2, 1\}$

$$E(\widehat{\Phi}_\lambda) = \begin{cases} 0, & \lambda = 0\,, \ \text{Perturbative Vacuum Solution,} \\ -1/2, & \lambda = 1/2\,, \ \text{Half Brane Solution,} \\ -1, & \lambda = 1\,, \ \text{Tachyon Vacuum Solution.} \end{cases} \tag{5.18}$$

Finally, we would like to evaluate the energy $E(\widehat{\Phi}_\lambda)$ for a generic value of the parameter λ. This computation will be performed in the next section.

6 Energy of the one-parameter family of solutions

In order to evaluate the energy associated to the one-parameter family of solutions $\widehat{\Phi}_\lambda$ for a generic value of the parameter λ, it will be useful to express the functions (4.6) and (4.7) as superpositions of wedge states $\Omega^t = e^{-tK}$, to this end, let us start by rewriting the solution (4.8) as follows

$$\widehat{\Phi}_\lambda = \Phi_{1,\lambda}\otimes\sigma_3 + \Phi_{2,\lambda}\otimes i\sigma_2, \tag{6.1}$$

where the $GSO(\pm)$ components $\Phi_{1,\lambda}$ and $\Phi_{2,\lambda}$ are given by

$$\Phi_{1,\lambda} = Q(Bc)f(K,\lambda) + pcf(K,\lambda) + qcGBcG\widetilde{f}(K,\lambda), \tag{6.2}$$
$$\Phi_{2,\lambda} = Q(Bc)G\widetilde{f}(K,\lambda) + pcG\widetilde{f}(K,\lambda) + qcGBcf(K,\lambda), \tag{6.3}$$

and

$$f(K,\lambda) = \frac{p^2 + wK}{(K-r_1)(K-r_2)}\,, \tag{6.4}$$
$$\widetilde{f}(K,\lambda) = \frac{qK}{(K-r_1)(K-r_2)}\,. \tag{6.5}$$

The set of parameters p, q, w, r_1 and r_2 have been defined as

$$p = \lambda(2\lambda-1), \qquad q = 4i\lambda(1-\lambda), \qquad w = \lambda(16\lambda^3 - 32\lambda^2 + 18\lambda - 1), \tag{6.6}$$
$$r_1 = -8\lambda^4 + 16\lambda^3 - 10\lambda^2 + \lambda - 4\sqrt{4\lambda^8 - 16\lambda^7 + 26\lambda^6 - 21\lambda^5 + 8\lambda^4 - \lambda^3}, \tag{6.7}$$
$$r_2 = -8\lambda^4 + 16\lambda^3 - 10\lambda^2 + \lambda + 4\sqrt{4\lambda^8 - 16\lambda^7 + 26\lambda^6 - 21\lambda^5 + 8\lambda^4 - \lambda^3}. \tag{6.8}$$

Using partial fraction decomposition, the functions defined by equations (6.4) and (6.5) can be expressed as

$$f(K,\lambda) = \frac{\alpha_1}{K-r_1} + \frac{\beta_1}{K-r_2}\,, \tag{6.9}$$
$$\widetilde{f}(K,\lambda) = \frac{\alpha_2}{K-r_1} + \frac{\beta_2}{K-r_2}\,, \tag{6.10}$$

where the parameters α_1, α_2, β_1 and β_2 are given by

$$\alpha_1 = \frac{p^2 + r_1 w}{r_1 - r_2}\,, \qquad \beta_1 = -\frac{p^2 + r_2 w}{r_1 - r_2}\,, \tag{6.11}$$
$$\alpha_2 = \frac{qr_1}{r_1 - r_2}\,, \qquad \beta_2 = -\frac{qr_2}{r_1 - r_2}\,. \tag{6.12}$$

The way how we have written the functions (6.9) and (6.10) allow us to represent these functions as the following integrals

$$f(K,\lambda) = \int_0^\infty dt \big[\alpha_1 e^{r_1 t} + \beta_1 e^{r_2 t}\big]\Omega^t\,, \tag{6.13}$$
$$\widetilde{f}(K,\lambda) = \int_0^\infty dt \big[\alpha_2 e^{r_1 t} + \beta_2 e^{r_2 t}\big]\Omega^t\,. \tag{6.14}$$

This integral representation constitutes a superposition of wedge states $\Omega^t = e^{-tK}$ [23].

In order for these integrals (6.13) and (6.14) to provide convergent results, we should require

$$\big(\Re r_1 < 0\big) \wedge \big(\Re r_2 < 0\big). \tag{6.15}$$

Using equations (6.7) and (6.8), from this inequality (6.15) we obtain the following conditions for the parameter λ

$$\big(\lambda < 0\big) \vee \left(\kappa \leq \lambda < \frac{1}{2}\right) \vee \left(\lambda > \frac{1}{2}\right), \tag{6.16}$$

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where κ is a numerical constant defined as

$$\kappa = \frac{2}{3} - \frac{1}{6}\left(\frac{25}{2} + \frac{3}{2}\sqrt{69}\right)^{1/3} - \frac{1}{6}\left(\frac{25}{2} - \frac{3}{2}\sqrt{69}\right)^{1/3} \approx 0.122561 \tag{6.17}$$

It is interesting to note that the region (6.16) does not contain the points $\lambda = 0$ and $\lambda = 1/2$ which corresponds to the perturbative vacuum and the half brane solution respectively. Physically this means that the various values: $\lambda = 0$, $\lambda = 1/2$ and the ones defined by the region (6.16) formally correspond to distinct gauge orbits within the formal solution (4.2).

Now, we are going to evaluate the energy $E(\widehat{\Phi}_\lambda)$ associated to a parameter λ belonging to the region (6.16). We might anticipate the result using the following argument. Due to the fact that the energy is a gauge invariant quantity, and since λ belonging to the region (6.16) corresponds to an specific gauge orbit, to compute the energy, we can choose a particular value for the parameter λ contained in this region, for instance $\lambda = 1$ which we know corresponds to the tachyon vacuum solution, therefore we should obtain the following result for the energy

$$E(\widehat{\Phi}_\lambda) = -1\,, \quad \text{for} \;\; (\lambda < 0) \vee \left(\kappa \leq \lambda < \frac{1}{2}\right) \vee \left(\lambda > \frac{1}{2}\right). \tag{6.18}$$

Employing the solution (6.1) together with the integral representation of the functions f and $\widetilde{f}$ given by (6.13) and (6.14), we would like to check the validity of the above result.

Using equation (4.13) for the case of the solution (6.1), and noting that the BRST exact terms do not contribute to the evaluation of the energy, we obtain

$$\begin{aligned} E(\widehat{\Phi}_\lambda) = \frac{\pi^2}{3}\Big[&p^2\langle\langle cfQ(c)f\rangle\rangle + q^2\langle\langle cGBcG\widetilde{f}Q(cGBc)G\widetilde{f}\rangle\rangle + 2pq\langle\langle cGBcG\widetilde{f}Q(c)f\rangle\rangle \\ &+p^2\langle\langle cG\widetilde{f}Q(c)G\widetilde{f}\rangle\rangle + q^2\langle\langle cGBcfQ(cGBc)f\rangle\rangle + 2pq\langle\langle cGBcfQ(c)G\widetilde{f}\rangle\rangle\Big]. \end{aligned} \tag{6.19}$$

Employing the identities (3.29)–(3.34), the correlation functions (5.3), (5.7)–(5.10), the integrals (6.13) and (6.14), we can evaluate all the correlation functions which will appear from the R.H.S. of (6.19). For instance, let us compute $\langle\langle cfQ(c)f\rangle\rangle$

$$\langle\langle cfQ(c)f\rangle\rangle = -\langle Y_{-2}cf(K,\lambda)\gamma^2 f(K,\lambda)\rangle = -\int_0^\infty dt_1 dt_2\, h(t_1)h(t_2)\langle Y_{-2}c\Omega^{t_1}\gamma^2\Omega^{t_2}\rangle, \tag{6.20}$$

where we have defined $h(t) = \alpha_1 e^{r_1 t} + \beta_1 e^{r_2 t}$. Using the correlation function (5.3), we can derive the correlator

$$\langle Y_{-2}c\Omega^{t_1}\gamma^2\Omega^{t_2}\rangle = \frac{(t_1+t_2)^2}{2\pi^2}. \tag{6.21}$$

Plugging (6.21) into equation (6.20) and performing the change of variables $t_1 \to uv$,

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$t_2 \to u - uv$, $\int_0^\infty dt_1 dt_2 \to \int_0^\infty du \int_0^1 dv\, u$, we get

$$\begin{aligned}\langle\langle cfQ(c)f\rangle\rangle &= -\int_0^\infty du \int_0^1 dv\, \frac{u^3\left(\alpha_1 e^{r_1 uv}+\beta_1 e^{r_2 uv}\right)\left(\alpha_1 e^{r_1(u-uv)}+\beta_1 e^{r_2(u-uv)}\right)}{2\pi^2} \qquad (6.22)\\ &= -\frac{2\alpha_1\beta_1 r_1 r_2(r_1^2+r_2 r_1+r_2^2)+3\alpha_1^2 r_2^4+3\beta_1^2 r_1^4}{\pi^2 r_1^4 r_2^4}\\ &= \frac{3-38\lambda+64\lambda^2-32\lambda^3}{\pi^2\lambda^2(2\lambda-1)^3}\,. \qquad (6.23)\end{aligned}$$

The integral (6.22) exists only when $\Re r_{1,2} < 0$, and for such r_1, r_2, this integral has the value shown in equation (6.23). Note that we have a singularity at $\lambda = 0$ and $\lambda = 1/2$, while in the case where λ belongs to the region $(0, \kappa)$, the expression (6.23) is clearly well-defined. Therefore aside from these two singular points, it seems that the result of the integral does not differentiate between different regions of λ. We wonder if the same phenomenon can happen for the remaining integrals coming from the rest of terms on the R.H.S. of equation (6.19).

It turns out that the expressions for the remaining integrals will not be as simple as the one shown in (6.22). For instance, from the second term on the R.H.S. of equation (6.19), after performing algebraic manipulations, we obtain a lot of terms and just as an illustration, let us show one of them

$$\mathcal{I}(\lambda) \equiv \langle Y_{-2}BKc\widetilde{f}(K,\lambda)\gamma K\widetilde{f}(K,\lambda)c\gamma\rangle. \qquad (6.24)$$

Using the integral representation (6.14), and defining the function $g(t) = \alpha_2 e^{r_1 t} + \beta_2 e^{r_2 t}$, we can write equation (6.24) as follows

$$\mathcal{I}(\lambda) = \int_0^\infty dt_1 dt_2\, g(t_1)g(t_2)\langle Y_{-2}BKc\Omega^{t_1}\gamma K\Omega^{t_2}c\gamma\rangle. \qquad (6.25)$$

Employing the correlation function (5.7), we can derive the correlator

$$\langle Y_{-2}BKc\Omega^{t_1}\gamma K\Omega^{t_2}c\gamma\rangle = \frac{\pi t_2\,(t_1+t_2)\sin\left(\frac{\pi t_2}{t_1+t_2}\right)+\left(t_1^2+\left(2+\pi^2\right)t_2 t_1+t_2^2\right)\cos\left(\frac{\pi t_2}{t_1+t_2}\right)}{2\pi^2\,(t_1+t_2)^2}. \qquad (6.26)$$

Plugging (6.26) into equation (6.25) and performing the change of variables $t_1 \to uv$, $t_2 \to u - uv$, $\int_0^\infty dt_1 dt_2 \to \int_0^\infty du \int_0^1 dv\, u$, the integral over the variable v can be easily done, so that we get

$$\mathcal{I}(\lambda) = \int_0^\infty du \frac{\alpha_2 u e^{r_1 u}\left(\beta_2 + \frac{u^2}{\pi^2}\alpha_2(r_1-r_2)^2+\alpha_2\right)+\beta_2 u e^{r_2 u}\left(\alpha_2+\frac{u^2}{\pi^2}\beta_2(r_1-r_2)^2+\beta_2\right)}{2\pi^2+2(r_1-r_2)^2 u^2}. \qquad (6.27)$$

The above integral exists only when $\Re r_{1,2} < 0$, and unlike the integral (6.22), here we were not able to write a simple analytic expression for the result of this integral (6.27). Nevertheless, for the parameter λ belonging to the region (6.16), integrals like (6.27) can

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be evaluated numerically with arbitrary precision. The numerical evaluation of these type of integrals blows up in the range where $\lambda \in (0, \kappa)$.

Carrying out similar computations for the rest of terms on the R.H.S. of equation (6.19), adding the results up and performing numerical integration[6] together with the definitions (6.6)–(6.8), (6.11), (6.12), the energy turns out to be

$$E(\widehat{\Phi}_\lambda) = \frac{\pi^2}{3}\Big[-0.303963550927\ldots \Big] = \frac{\pi^2}{3}\Big[-\frac{3}{\pi^2}\Big] = -1. \tag{6.28}$$

Collecting the results (5.18) and (6.28), we can summarize the main result of our paper

$$E(\widehat{\Phi}_\lambda) = \begin{cases} 0, & \lambda = 0\,, \;\; \text{Perturbative Vacuum Solution,} \\ -1/2, & \lambda = 1/2\,, \;\; \text{Half Brane Solution,} \\ -1, & \big(\lambda < 0\big) \vee \big(\kappa \leq \lambda < \tfrac{1}{2}\big) \vee \big(\lambda > \tfrac{1}{2}\big)\,, \;\; \text{Tachyon Vacuum Solution,} \end{cases} \tag{6.29}$$

namely, depending on the value of the parameter λ, the solution represents three distinct gauge orbits corresponding to the perturbative vacuum, the half brane and the tachyon vacuum solution.

7 Summary and discussion

We have studied and constructed a one-parameter family of solutions which contains the perturbative vacuum, the half brane and the tachyon vacuum solution in the modified cubic superstring field theory. To our knowledge, this is the first explicit example of a solution which describes these three distinct gauge orbits.

To evaluate the energy associated to the one-parameter family of solutions we have performed analytic computations, however it would be nice to confirm our results by employing numerical techniques such as the curly $\mathcal{L}_0$ level expansion [30–32] or the usual Virasoro L_0 level expansion scheme [33–35]. The numerical analysis should be important, for instance, to check if the solution behaves as a regular element in the state space constructed out of Fock states [23, 36, 37].

In the case of open bosonic string field theory, using elements of the KBc subalgebra, in reference [17], the existence of physically distinct solutions has been analyzed such as the perturbative vacuum, the tachyon vacuum and the MNT ghost brane [38, 39]. Following the lines developed in this paper, it would be nice to find a one-parameter family of solutions which describes these distinct gauge orbits.

Finally, we would like to comment that the construction of solutions based on gauge transformation of identity based solutions can be generalized in order to consider more cumbersome solutions, such as the multibrane solutions [19, 20], and the recently proposed Erler's analytic solution for tachyon condensation in Berkovits open superstring field theory [24]. Since the algebraic structure of Berkovits theory [25] is similar to the cubic superstring field theory, the results of our work can be naturally extended, however the presence of a non-polynomial action will bring us challenges in the search of new solutions within Berkovits theory.

[6]The explicit expression for the result of the energy in terms of integrals over the variable u is shown in appendix A.

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A Explicit expression for the energy of the one-parameter family of solutions

Here we are going to write the explicit expression for the energy of the one-parameter family of solutions derived from the evaluation of all terms on the R.H.S. of equation (6.19). The result reads as follows

$$E(\widehat{\Phi}_\lambda) = \frac{\pi^2}{3}\Big[p^2\mathcal{I}_1(\lambda) + pq\mathcal{I}_2(\lambda) + q^2\mathcal{I}_3(\lambda)\Big], \tag{A.1}$$

where the functions $\mathcal{I}_1(\lambda)$, $\mathcal{I}_2(\lambda)$ and $\mathcal{I}_3(\lambda)$ are given by

$$\begin{aligned}\mathcal{I}_1(\lambda) &= \frac{-32\lambda^3 + 64\lambda^2 - 38\lambda + 3}{\pi^2\lambda^2(2\lambda-1)^3} \\ &\quad + \int_0^\infty du \frac{2u^2\Big[\alpha_2 e^{r_1 u}\big(\beta_2 + \frac{u^2}{\pi^2}\alpha_2(r_1-r_2)^2+\alpha_2\big)+\beta_2 e^{r_2 u}\big(\alpha_2+\frac{u^2}{\pi^2}\beta_2(r_1-r_2)^2+\beta_2\big)\Big]}{\pi^2+(r_1-r_2)^2u^2}.\end{aligned} \tag{A.2}$$

$$\begin{aligned}\mathcal{I}_2(\lambda) = \int_0^\infty du\, &\frac{u}{\pi^2\left(\pi^2+u^2\left(r_1-r_2\right)^2\right)^2\left(r_1-r_2\right)}\Big[\\ &(-e^{ur_1}\left(\alpha_2\left(3ur_2\left(\pi^2+u^2r_2^2\right)^2\alpha_1+\left(\pi^4+3u^4r_2^4\right)\beta_1\right)\right. \\ &+\left(\pi^4+ur_2\left(4\pi^4+ur_2\left(4\pi^2+ur_2\left(4\pi^2-ur_2\right)\right)\right)\right)\alpha_1\beta_2) \\ &+e^{ur_2}\left(\left(\pi^4-ur_2\left(4\pi^4+ur_2\left(-4\pi^2+ur_2\left(6\pi^2+ur_2\left(1+2ur_2\right)\right)\right)\right)\right)\alpha_2\beta_1\right. \\ &+\left(\left(\pi^4+u^3r_2^3\left(2\pi^2+ur_2\left(3+2ur_2\right)\right)\right)\alpha_1-3ur_2\left(\pi^2+u^2r_2^2\right)^2\beta_1\right)\beta_2) \\ &+u^4r_1^4\left(e^{ur_1}\left(\alpha_2\left(-3\beta_1+ur_2\left(-15\alpha_1+8\beta_1\right)\right)+\left(1-8ur_2\right)\alpha_1\beta_2\right)\right. \\ &-e^{ur_2}\left(\left(1+2ur_2\right)\alpha_2\beta_1+\left(-\left(3+2ur_2\right)\alpha_1+15ur_2\beta_1\right)\beta_2\right)) \\ &+2u^2r_1^2\left(-e^{ur_1}\left(ur_2\alpha_2\left(3\left(3\pi^2+5u^2r_2^2\right)\alpha_1+\left(-2\pi^2+ur_2\left(9-4ur_2\right)\right)\beta_1\right)\right.\right. \\ &+\left(2\pi^2+ur_2\left(8\pi^2+ur_2\left(-3+4ur_2\right)\right)\right)\alpha_1\beta_2) \\ &+e^{ur_2}\left(-\left(-2\pi^2+ur_2\left(7\pi^2+3ur_2\left(1+2ur_2\right)\right)\right)\alpha_2\beta_1\right. \\ &+ur_2\left(\left(\pi^2+3ur_2\left(3+2ur_2\right)\right)\alpha_1-3\left(3\pi^2+5u^2r_2^2\right)\beta_1\right)\beta_2)) \\ &+u^5r_1^5\left(3e^{ur_2}\beta_1\beta_2+e^{ur_1}\left(-2\alpha_2\beta_1+\alpha_1\left(3\alpha_2+2\beta_2\right)\right)\right) \\ &+2u^3r_1^3\left(e^{ur_2}\left(2\left(\pi^2+ur_2\left(1+2ur_2\right)\right)\alpha_2\beta_1+\left(-2ur_2\left(3+2ur_2\right)\alpha_1\right.\right.\right. \\ &+3\left(\pi^2+5u^2r_2^2\right)\beta_1)\beta_2)+e^{ur_1}\left(-\left(\pi^2+6ur_2\left(-1+ur_2\right)\right)\alpha_2\beta_1\right. \\ &+\alpha_1\left(3\left(\pi^2+5u^2r_2^2\right)\alpha_2+\left(3\pi^2+2ur_2\left(-1+3ur_2\right)\right)\beta_2\right)))\end{aligned}$$

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$$
\begin{aligned}
&+ur_1\left(e^{ur_2}\left(4\left(\pi^4+ur_2\left(-2\pi^2+ur_2\left(4\pi^2+ur_2\left(1+2ur_2\right)\right)\right)\right)\alpha_2\beta_1\right.\right.\\
&+\left(-4u^2r_2^2\left(\pi^2+ur_2\left(3+2ur_2\right)\right)\alpha_1+3\left(\pi^4+6\pi^2u^2r_2^2+5u^4r_2^4\right)\beta_1\right)\beta_2\right)\\
&+e^{ur_1}\left(-2u^2r_2^2\left(\pi^2+ur_2\left(-6+ur_2\right)\right)\alpha_2\beta_1+\alpha_1\left(3\left(\pi^4+6\pi^2u^2r_2^2+5u^4r_2^4\right)\alpha_2\right.\right.\\
&\left.\left.\left.\left.+2\left(2\pi^4+ur_2\left(4\pi^2+ur_2\left(7\pi^2+ur_2\left(-2+ur_2\right)\right)\right)\right)\beta_2\right)\right)\right)\right)\Bigg].
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
&\mathcal{I}_3(\lambda)=\int_0^\infty du\ \frac{u}{2\pi^4\left(\pi^2+u^2\left(r_1-r_2\right)^2\right)^2\left(r_1-r_2\right)}\Bigg[\\
&\left(e^{ur_1}\left(8+\pi^2\right)u^6r_1^7\alpha_2^2+e^{ur_1}\pi^2\left(-3r_2\left(\pi^2+u^2r_2^2\right)^2\alpha_2^2-2\left(\pi^4+3u^4r_2^4\right)\alpha_1\beta_1\right)\right.\\
&+e^{ur_1}u^5r_1^6\left(\left(-8+\pi^2\right)u\alpha_1^2+\alpha_2\left(\left(9\pi^2-5\left(8+\pi^2\right)ur_2\right)\alpha_2+6\pi^2\beta_2\right)\right)\\
&+ur_1^2\left(e^{ur_1}\left(-8+\pi^2\right)u\left(\pi^4+6\pi^2u^2r_2^2+5u^4r_2^4\right)\alpha_1^2-e^{ur_1}\right.\\
&\left(-9\pi^6+ur_2\left(\pi^4\left(26+\pi^2\right)+ur_2\left(-54\pi^4+ur_2\left(2\pi^2\left(23+\pi^2\right)+ur_2\left(-45\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\right)\right)\right)\\
&\alpha_2^2+4\pi^2u\left(-2e^{ur_1}\pi^2+u^2r_2^2\left(3e^{ur_2}\left(1+ur_2\right)+e^{ur_1}\left(-3+2ur_2\right)\right)\right)\alpha_1\beta_1\\
&+2\pi^2\left(e^{ur_1}\left(\pi^2+3u^2r_2^2\right)\left(3\pi^2+ur_2\left(-8+ur_2\right)\right)-2e^{ur_2}ur_2\left(\pi^2+ur_2\left(5\pi^2+3ur_2\left(5+3ur_2\right)\right)\right)\right)\\
&\alpha_2\beta_2-2e^{ur_2}ur_2\left(3\pi^2+5u^2r_2^2\right)\left(\left(-8+\pi^2\right)u^2r_2\beta_1^2+\left(3\pi^2+ur_2\left(9\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\beta_2^2\right)\Big)\\
&-e^{ur_2}\left(2\pi^2\left(-\pi^4+ur_2\left(\pi^4-4\pi^2ur_2+u^3r_2^3-u^4r_2^4\right)\right)\alpha_1\beta_1+r_2\left(\pi^2+u^2r_2^2\right)^2\right.\\
&\left.\left(\left(-8+\pi^2\right)u^2r_2\beta_1^2+6\pi^2\left(1+ur_2\right)\alpha_2\beta_2+\left(3\pi^2+ur_2\left(9\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\beta_2^2\right)\right)\\
&+u^3r_1^4\left(2e^{ur_1}\left(-8+\pi^2\right)u\left(\pi^2+5u^2r_2^2\right)\alpha_1^2+2\pi^2u\left(e^{ur_2}\left(3+ur_2\right)+e^{ur_1}\left(1+4ur_2\right)\right)\alpha_1\beta_1\right.\\
&-e^{ur_1}\alpha_2\left(\left(-18\pi^4+ur_2\left(63\pi^2+6\pi^4+10ur_2\left(-9\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\right)\alpha_2\right.\\
&\left.-4\pi^2\left(3\pi^2+ur_2\left(-8+9ur_2\right)\right)\beta_2\right)\\
&\left.-e^{ur_2}ur_2\left(5\left(-8+\pi^2\right)u^2r_2\beta_1^2+2\pi^2\left(7+3ur_2\right)\alpha_2\beta_2+5\left(3\pi^2+ur_2\left(9\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\beta_2^2\right)\right)\\
&+u^4r_1^5\left(e^{ur_2}\left(\left(-8+\pi^2\right)u^2r_2\beta_1^2+\left(3\pi^2+ur_2\left(9\pi^2+\left(8+\pi^2\right)ur_2\right)\right)\beta_2^2\right)+e^{ur_1}\left(10\left(8+\pi^2\right)u^2r_2^2\alpha_2^2\right.\right.\\
&\left.\left.+\pi^2\left(\left(19+2\pi^2\right)\alpha_2^2-2u\alpha_1\beta_1+6\alpha_2\beta_2\right)-ur_2\left(5\left(-8+\pi^2\right)u\alpha_1^2+3\pi^2\alpha_2\left(15\alpha_2+8\beta_2\right)\right)\right)\right)\\
&+u^2r_1^3\left(5\left(8+\pi^2\right)u^4r_2^4\left(e^{ur_1}\alpha_2^2+2e^{ur_2}\beta_2^2\right)+\pi^4\left(6e^{ur_2}\beta_2^2+e^{ur_1}\alpha_2\left(\left(14+\pi^2\right)\alpha_2+12\beta_2\right)\right)\right.\\
&+2\pi^2ur_2\left(e^{ur_2}\left(-8u\alpha_1\beta_1+\left(-8+\pi^2\right)u\beta_1^2+\pi^2\beta_2\left(2\alpha_2+9\beta_2\right)\right)\right.\\
&\left.-e^{ur_1}\left(3\left(-8+\pi^2\right)u\alpha_1^2+\pi^2\alpha_2\left(27\alpha_2+14\beta_2\right)\right)\right)+2u^3r_2^3\left(-e^{ur_1}\right.\\
&\left.\left(5\left(-8+\pi^2\right)u\alpha_1^2+3\pi^2\alpha_2\left(15\alpha_2+4\beta_2\right)\right)+e^{ur_2}\left(5\left(-8+\pi^2\right)u\beta_1^2+3\pi^2\beta_2\left(4\alpha_2+15\beta_2\right)\right)\right)+2\pi^2\\
&\left.u^2r_2^2\left(3e^{ur_1}\left(\left(13+\pi^2\right)\alpha_2^2-2u\alpha_1\beta_1+10\alpha_2\beta_2\right)+e^{ur_2}\left(-4u\alpha_1\beta_1+\beta_2\left(24\alpha_2+\left(23+\pi^2\right)\beta_2\right)\right)\right)\right)\\
&+r_1\left(5e^{ur_2}\left(8+\pi^2\right)u^6r_2^6\beta_2^2+\pi^4u^2r_2^2\left(2e^{ur_1}\alpha_2\left(9\alpha_2+2\beta_2\right)+e^{ur_2}\beta_2\left(16\alpha_2+\left(26+\pi^2\right)\beta_2\right)\right)\right.\\
&+\pi^6\left(3e^{ur_2}\beta_2^2+e^{ur_1}\left(2u\alpha_1\beta_1+3\alpha_2\left(\alpha_2+2\beta_2\right)\right)\right)\\
&+\pi^4ur_2\left(-e^{ur_1}\left(\left(-8+\pi^2\right)u\alpha_1^2-8u\alpha_1\beta_1+\pi^2\alpha_2\left(9\alpha_2+4\beta_2\right)\right)\right.\\
&\left.+e^{ur_2}\left(-8u\alpha_1\beta_1+\left(-8+\pi^2\right)u\beta_1^2+\pi^2\beta_2\left(4\alpha_2+9\beta_2\right)\right)\right)\\
&+u^5r_2^5\left(-e^{ur_1}\left(\left(-8+\pi^2\right)u\alpha_1^2+9\pi^2\alpha_2^2\right)+e^{ur_2}\left(5\left(-8+\pi^2\right)u\beta_1^2+3\pi^2\beta_2\left(8\alpha_2+15\beta_2\right)\right)\right)\\
&+2\pi^2u^3r_2^3\left(-e^{ur_1}\left(\left(-8+\pi^2\right)u\alpha_1^2-8u\alpha_1\beta_1+\pi^2\alpha_2\left(9\alpha_2+2\beta_2\right)\right)\right.\\
&\left.+e^{ur_2}\left(3\left(-8+\pi^2\right)u\beta_1^2+\pi^2\beta_2\left(14\alpha_2+27\beta_2\right)\right)\right)\\
&\left.\left.+\pi^2u^4r_2^4\left(e^{ur_1}\left(15\alpha_2^2-2u\alpha_1\beta_1+14\alpha_2\beta_2\right)+e^{ur_2}\left(-8u\alpha_1\beta_1+\beta_2\left(32\alpha_2+3\left(21+2\pi^2\right)\beta_2\right)\right)\right)\right)\right)\Bigg].
\end{aligned}
\tag{A.4}
$$

The above integrals converge provided that $\Re\, r_{1,2} < 0$, and can be computed numerically with arbitrary precision.

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